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DISCUSSION OF PRACTICAL DESIGN OF SOLID-BARREL, REINFORCED-CONCRETE SKEW STRUCTURES

(*Published in October, 1950*)

By Maurice Barron, Arthur Hayden, and
Bernard L. Weiner

STRUCTURAL DIVISION

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DISCUSSION

Maurice Barron,[9] M. ASCE.—In several respects, this paper corroborates and confirms the methods, procedures, and conclusions of a previous paper and discussion on this subject by the writer. It is interesting to compare the conclusions and arguments presented by the author with similar items in the writer's paper:

1a (Weiner)

"1. With more than sufficient accuracy for practical design purposes, the 'basic stresses' and the 'skew stresses' (total) may be separated and treated as separate elastic systems."

1b (Barron)[10]

"It has been demonstrated that for all practical design purposes:

"1. With few exceptions, the rectangular elastic system is independent of the torsional elastic system;

"2. The rectangular redundants and the rectangular stresses for all applied loads are independent * * * of the skew angle; * * *"

2a (Weiner)

"2. The basic stresses, except for temperature and shrinkage, are very closely equal to those in a similar right-angle structure with the same square span and with the same elastic properties. For temperature and shrinkage, the basic stresses obtained from the same right-frame analysis are multiplied by the square of the secant of the skew angle."

2b (Barron)[10]

"5. The rectangular redundants due to temperature change vary as the sec^2—that is, as $(1 + \epsilon^2)$; * * *

"6. The torsional stresses may be written directly from the rectangular stresses by multiplying an appropriate rectangular stress by an appropriate geometric or elastic constant, thereby completely eliminating the torsional elastic analysis; * * *"

3a (Weiner)

"* * * It will be shown that, in most cases, the 'skew stresses' need not be computed at all for design purposes. Designs that are adequate for both safety and economy can be made by applying a rather simple correction to a right-frame design.

"The right-frame analysis for the total stresses can be made by any of the methods in common use and this phase of the problem, except for one detail, therefore, needs no further discussion * * *"

3b (Barron)[10]

"12. For practical office procedure no torsional analysis is required, and the longitudinal steel may be determined from the rectangular design using the 'turning process'; * * *

"10. Any method that will determine the rectangular stresses satisfactorily may be used as the basis for determining the torsional stresses (such other methods include moment distribution and slope deflection); * * *"

Note.—This paper by Bernard L. Weiner was published in October, 1950, as *Proceedings-Separate No. 39*. The numbering of footnotes, illustrations, and equations are a continuation of the consecutive numbering used in the original paper.

[9] Cons. Engr., Farkas & Barron, New York, N. Y.

[10] "Reinforced Concrete Skewed Rigid-Frame and Arch Bridges," by Maurice Barron, *Proceedings-Separate No. 13*, ASCE, April, 1950.

4a (Weiner)

"It has been found in the general case, however, that, where the neutral surfaces of the several members are neither straight nor vertical and horizontal, there is little error in considering the two sets of stresses independently and in analyzing them as two separate elastic systems. * * * The approximation holds good for any number of spans and for any shape of frame or arch that would be encountered in practical design."

4b (Barron)[11]

"* * * The evidence shows that the two elastic systems of single-span skewed structures are practically independent for vertical and horizontal loads and equally independent of the shape of the structure. A double-span structure may be considered as a single span with an elastic bracket at the center, to the ends of which are applied forces equal to the redundant reactions. This logic may be extended to include three or more spans."

There are several other significant points of agreement in the two papers. It should be noted that the author states his proposed method has been checked by comparing the theoretical results with previous designs, and the presentation of substantiating data would be helpful. For example, the opening statement of the paper is "No new theories are presented in this paper." As a matter of fact the paper makes use of several supplementary sources of new information and ideas, several of which are not properly cited.

There are three contradictions in this paper which should be reconciled so that an inexperienced reader will not be confused:

5a.—In the third paragraph of the "Introduction"—

> "For 'small' skews, the expedient of designing on the skew span has often been used. A consideration of the arguments in this paper will show that this expedient is entirely incorrect; it leads to results that either are on the side of danger or are very wasteful."

whereas (item 5b), in the fourth paragraph—

> "The thickness of the members and the amount of reinforcing steel do not appear excessive or unusual compared to that required for a right-angle structure of span equal approximately to the skew span. Such skew structures have been built and have been in service from 6 years to 20 years without showing signs of distress."

6a.—In the first paragraph of "Characteristics of Skew Frames"—

> "The practical design of most structures—as distinct from the theoretical analysis for research purposes—may be divided into two steps:
>
> 1. The computation of the total moments, thrusts, and shears; and
> 2. The design of the sections to resist the total stresses computed in step 1."

whereas (item 6b) in the second paragraph—

> "In the case of wide structures such as slabs, total stresses have little significance and are practically useless for design purposes."

[11] Discussion by Maurice Barron of "Simplified Analysis of Skewed Reinforced Concrete Frames and Arches," by Richard M. Hodges, *Transactions*, ASCE, Vol. 109, 1944, p. 966.

7a.—In the last paragraph of "Characteristics of Skew Frames"—

> "In theory, then, the skew frame is treated as a whole in three dimensions, and dividing by b is merely an arithmetical convenience."

whereas (item 7b), in the paragraph immediately preceding—

> "It is found, however, that the skew width b, measured parallel to the abutments, can be factored out of all the terms of the equations but one, the so-called 'factor of torsion.'"

A comparison of the contents of other papers and discussions with the contents of this paper reveals a repetition of contradictions that have caused confusion since the early 1930's. The author can perform a very worthy service at this time by reconciling the conflicts. The following four instances are indicative:

8a.—In the paragraphs following Eq. 23, Mr. Weiner states:

> "* * * the fascia section extends for a distance of three or four times the depth of the section * * * the maximum section of the fascia should be used for the entire width."

In 1931, he advocated[12] a fascia extension of only one and one-half times the depth of the sections (item 8b):

> "The fascia section extends for a distance (parallel to the abutments) of about one and one-half times the depth of the section; and where cross-steel is required, it should extend for at least a bond length of the cross-steel."

What are the reasons for the wider fascia?

9a.—Under the heading "Stresses in the Interior Parts of a Section," Mr. Weiner states that:

> "* * * the design of the fascia, only, is sufficient. * * * designs, for single-span and multiple-span frames and full centered 'arches' (frames with curved soffits) and for skew angles up to 50°, also showed that adequate designs can be made without recourse to complete skew-arch analysis."

yet (item 9b), in his discussion of the paper by Richard M. Hodges[13] he implies quite the opposite.

10a.—In the last paragraph of "Stresses in the Interior Parts of a Section," Mr. Weiner recommends that:

> "* * * the larger the skew, the greater the relative thickness of concrete should be; that is, as the skew increases, less compression reinforcement should be used, thus resulting in thicker crown sections. Skews of much more than 50° become impractical for purely geometric reasons; and, for this upper limit, it seems wise to make the crown sufficiently deep so that it can be designed without any compression steel. Although the single-span frame with a 50° skew, previously mentioned, was designed with

[12] "Design of a Reinforced Concrete Skew Arch," by Bernard L. Weiner, *Transactions*, ASCE, Vol. 96, 1932, p. 1274.

[13] Discussion by Bernard L. Weiner of "Simplified Analysis of Skewed Reinforced Concrete Frames and Arches," by Richard M. Hodges, *Transactions*, ASCE, Vol. 109, 1944, pp. 948 and 953.

rather heavy compression reinforcement and showed no signs of distress after almost 20 years of service, the practice is not recommended—unless a complete design is made."

This does not "square" (item 10b) with the logic advanced by Phillips H. Lovering, Assoc. M. ASCE, in his discussion[14] of the Hodges paper nor with

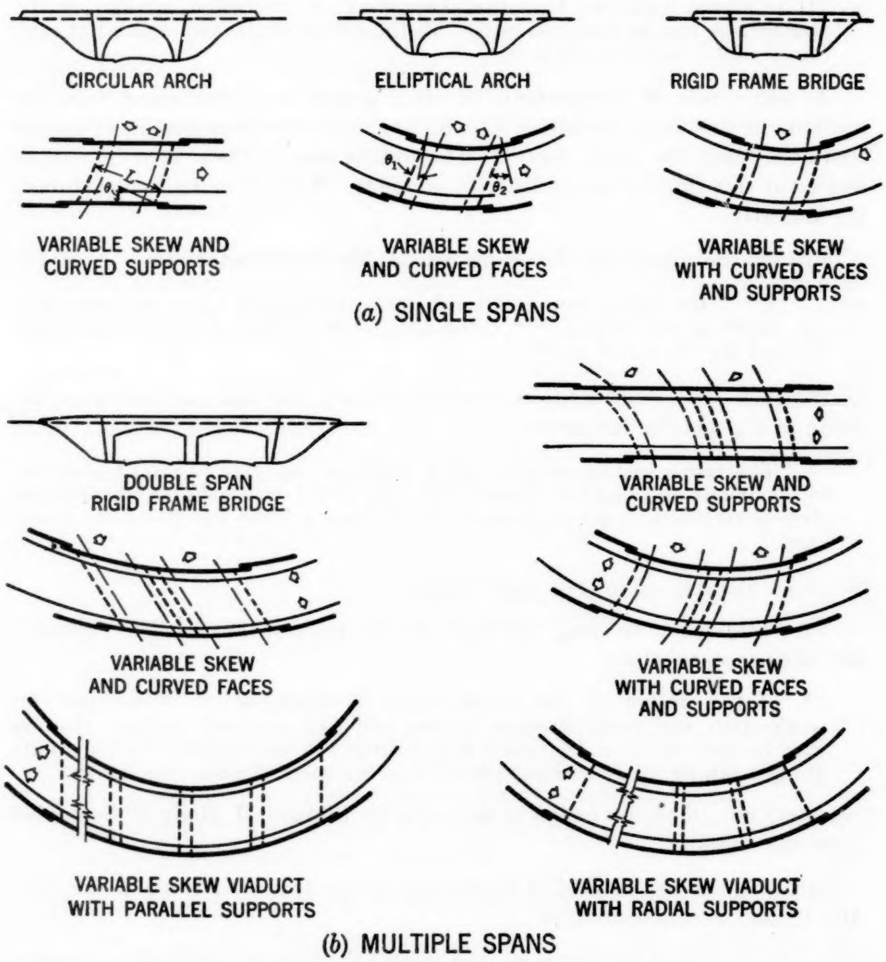

FIG. 9

Mr. Hodges' reply emphasizing the fact that temperature effects for structures with large skew are the controlling factors and that "it is best to thicken the knee."

11a.—Finally, under the heading "Characteristics of Skew Frames," Mr. Weiner declares:

[14] Discussion by Phillips H. Lovering of "Simplified Analysis of Skewed Reinforced Concrete Frames and Arches," by Richard M. Hodges, *Transactions*, ASCE, Vol. 109, 1944, p. 966.

"The analysis and the design of a rectangular frame or arch, for example, are really based on the assumption that the frame or arch can be divided into narrow strips, each one of which may be designed as a narrow beam—because of symmetry along the width of the structure."

which is quite inconsistent with his comparable statement[15] (under the heading "General Remarks") in discussing the Hodges paper (item 11b):

"* * * In spite of the misleading statements that have been made time and time again, the skew arch must be analyzed as a complete unit and the division by b is merely a minor arithmetic convenience which has no structural significance."

Concerning item 11, a little different viewpoint will clear up the misunderstanding concerning an analysis or design using 1-ft strips. Neither the right nor skewed structure is really considered, by most designers, to be cut into strips. The structure is considered in its entirety. Since the width of the structure is a linear function in every stress equation and in every load equation, using values per foot of width avoids the use of large numbers, permits the use of published design aids, and helps the designer in many other ways.

Until the writer demonstrated[10] that the rectangular elastic system is almost independent of the torsional elastic system and was able to determine the effects of each term quantitatively in the "exact" analysis, the effects of variable skew could not be determined, and these structures could not be analyzed, designed, or constructed. In his treatment of these commercially patented structures, Mr. Weiner does not show how the variable skew affects the foundation restraints or how total stresses are determined. Boundary design for the maximum skew can be shown to be inadequate. Fig. 9 shows several types of variable skewed curved-in-plan structures. Other combinations of variable skew, variable width, and variable span are shown elsewhere.[16]

There are few designs available for either single-span or double-span heavily skewed structures based on exact analysis. One of these rare specimens is a double-span structure designed under the author's supervision, which involved the simultaneous solution of eighteen elastic equations (three sets of six equations each). The author's double-span analysis has been used as a "guinea pig" on which to check the conclusions stated in the writer's paper.[10] In discussing these comparisons with the author, the writer requested an application of the same checks to his conclusions. The use of elastic weights[16] as advocated in this paper is a departure from the author's former preference[17,7,18] for the method of partial summation. The author repeatedly stresses his preference for "ellipse of stress" theory. For sections with large ratios of b/t, Mr. Rathbun has pointed out that the ellipse degenerates into

[15] Discussion by Bernard L. Weiner of "Simplified Analysis of Skewed Reinforced Concrete Frames and Arches," by Richard M. Hodges, *Transactions*, ASCE, Vol. 109, 1944, p. 953.

[16] "The Rigid-Frame Bridge," by Arthur G. Hayden and Maurice Barron, John Wiley & Sons, Inc., New York, N. Y., 1950.

[17] "Design of a Reinforced Concrete Skew Arch," by Bernard L. Weiner, *Transactions*, ASCE, Vol. 96, 1932, p. 1212.

[7] Discussion by B. L. Weiner of "An Analysis of Multiple-Skew Arches on Elastic Piers," by J. Charles Rathbun, *Transactions*, ASCE, Vol. 98, 1933, p. 46.

[18] Discussion by Bernard L. Weiner of "Simplified Analysis of Skewed Reinforced Concrete Frames and Arches," by Richard M. Hodges, *Transactions*, ASCE, Vol. 109, 1944, p. 946.

a straight line or into an extremely flat ellipse. Mr. Hodges and others have also shown that this refinement is not warranted.

The determination of the effect of re-entrance (negative slope of legs) in rigid-frame bridges is also a refinement that is not warranted. It is common knowledge that the assumed median line does not truly represent the structure as a line diagram. Some tables of the paper take the re-entrant effect into account, and other tables neglect this effect. This is an unnecessary complication and will be confusing to all but the experienced designers.

In the computations, different sections for analysis and design are used. By using the same section for both phases of the work several steps can be eliminated. This will result in a simpler and more practical office design. For example, Col. 6 of Table 2 need not be scaled influence ordinates but the $(H\ F)$-values could be copied directly from Table 1, providing Table 1 was also simplified by omitting Col. 12 and finding the elastic moment at the point of application of the load rather than between loads.

The division of axes can be improved by having a point at the center line of the frame, a critical design section.

The shear increment method for finding moments due to elastic loads or to applied loads has not been clearly demonstrated. In Table 1 values in Cols. 10, 11, and 12 should be placed halfway between points because they are shears and moment increments between points. The statement, that in the "* * * elastic load method, the 'elastic moment' at any point is the deflection at that point," is not complete. What is the deflection, in what direction, and due to what load? A more complete explanation and better illustration of the elastic weight method is shown elsewhere.[16]

The computations are much simpler, with less chance of error, if fixed loads such as dead loads, and earth pressure are considered as a single system and not as superimposed individual loads. The simple-span moments may be determined by shear increment method and the moments due to the total redundant may be added to find the final value.

No proofs or data are presented to justify a generalization that the approximate method proposed "* * * holds good for any number of spans or for any shape of frame or arch that would be encountered in practical design."

In Table 3, the definition of M_s as the "simple beam" moment is incorrectly applied. The re-entrant structure used is far from a simple beam. The same principle applies to Table 5 and Fig. 6. The peaks of the influence lines for simple spans are not at the same position as shown in the analysis.

An error in design procedure is the result of applying the transformation which has been derived for external forces to the unit stresses. In Table 9, it should be noted that K_1 for a balanced section is applied both to the reinforcing and to the concrete. The statement that $C_2 = T_2 = \frac{M_2}{2\,r}$ is incorrect. This equation is correct for T_2 but incorrect for C_2. Since moments and thrusts have been turned as shown in Cols. 3 and 4, it is incorrect to determine compressive stress in the concrete from these turned moments. In other words, vectorally adding a unit compressive stress and a unit shear stress in

concrete is incorrect. Using the author's method, if the reinforcement is placed perpendicular to the abutments, a different value for C_2 would result.

Eq. 25 is given without proof and is based upon a demonstration presented elsewhere. This formula expresses an approximation applicable to no other structure but an idealized frame with vertical legs and horizontal vault. The author presents no way of evaluating the error or approximation for arches or for high-vaulted rigid frames. It is improper to generalize Eq. 25 as is done in this paper. By including the effect of the reinforcement in the moment of inertia in the example, another unnecessary refinement has been added: I and E always appear as a product $(E\,I)$ and the uncertainties in evaluating E do not justify great refinement in I.

The writer cannot agree with the author's estimate of the moment at the bottom of the footing of a rigid-frame bridge being equal to 40% of the moment at the point 6. The vertical reaction of the example is approximately 30 kips; 40% of the moment at point 6, divided by 30 kips, gives an eccentricity of more than 5 ft to the foundation reaction. Unless the footing is anchored to rock so as to develop tension within its base,[19] this is an impossibility. The writer's investigation of several structures founded on soil results in an eccentricity of less than 2 in.

By extending the mathematical equations into the design phase, it has been shown[10] that the required steel areas are functions of the stresses and that the skew correction can be applied to the final answer rather than to the stresses as represented by Eqs. 18, 19, 20, and 21. When numerical values are substituted prematurely, the mathematical and structural significance is lost. Concerning Eq. 18, it should be observed that the first and last terms are insignificant when compared with the middle term and that for all practical design purposes

$$t_s = \epsilon' \frac{t_v}{b} \qquad (27)$$

Also, in Eq. 20*b*, $m_v = \epsilon' m_s$ and, therefore,[10] Eq. 20*b* reduces to Eq. 20*a*.

The statement that "* * * horizontal shears * * * are more in the nature of column shears, they are of no serious consequence" is questionable. Shearing stresses on a design section result from the cross shear, t_s, the radial shear, t_u, and the torsion moment, m_v. For a design section of a skewed bridge, the shearing stresses are the result of external forces and are not the shearing stresses on a plane oblique with the axial column load. For structures with large skew, these shear stresses have exceeded 1,000 lb per sq in., twice the maximum value stated by the author. The writer contends[10] that these stresses are frequently the controlling stresses regardless of how much reinforcement is provided.

Mr. Westergaard's equations, recommended in the section on transverse reinforcement, apply only to live loads on flat plates of uniform thickness. They do not consider earth pressure or temperature change, arch action, or the variable I. The justification for using these equations in determining transverse reinforcement is very obscure.

[19] "The Rigid-Frame Bridge," by Arthur G. Hayden and Maurice Barron, John Wiley & Sons, Inc., New York, N. Y., 1950, p. 42.

The conclusions stated in this paper are generally correct, but it is improper and unnecessary to generalize the problem of skewed structures based on a few numerical comparisons of "exact" and approximate designs or upon equations derived for an idealized rigid frame. If a better method were not available, such comparisons and generalizations would be proper. However, it has been shown[10] how a complete separation of the rectangular and torsional systems may be affected and how each skew effect may be evaluated as a secondary quantity. The complexity associated with the analysis and design of skew arches and rigid-frame bridges is a self-imposed complexity.

Arthur G. Hayden,[20] M. ASCE.—It appears that structural engineers are at the end of the long trail leading to the clarification and simplification of the analysis of skewed rigid-frame bridges. The writer wrestled with this problem unsuccessfully in the 1920's, and it remained for Richard M. Hodges[4] to take the first long step in this direction. However, neither he nor the writer nor anyone else succeeded in separating the skew effects so that they could be evaluated separately, thus avoiding lengthy tabulations and the solution of complicated systems of simultaneous equations. It remained for Maurice Barron to do this, and he succeeded; but his conclusions[21] were at first controversial and met with opposition for some time. His missionary work converted one sceptic after another before his conclusions were finally accepted, and it is gratifying that Mr. Weiner, in his restatement of the problem, corroborates Mr. Barron in every respect.

As a result of these labors in the field of structural analysis the ordinary designer of bridges for grade separations has a power that he did not formerly possess.

Bernard L. Weiner,[22] M. ASCE.—In scientific research the "work" is the important thing; who did what, is of minor importance. However, since both Messrs. Barron and Hayden—the former in great detail—attempt to show how various writers have reached more or less the same conclusions, it would be well to consider the chronology of events briefly.

The present paper is an extension of a previous paper by the writer published in 1932.[23] Part III of this earlier paper shows mathematically that the two elastic systems can be separated in one type of structure—where the median lines of the members of a single-span frame are straight and perpendicular to each other. It also shows that for the more general type of single-span frame in which the members are neither straight nor perpendicular to each other, separation of the two elastic systems results in very little error. This analysis forecasts the possibility of extending the approximation to full-centered arches, to high-vaulted rigid frames, and to multiple spans. The writer has since analyzed such skew structures and found the same approximations to be valid.

[20] St. Michaels, Md.

[4] "Simplified Analysis of Skewed Reinforced Concrete Frames and Arches," by Richard M. Hodges, *Transactions*, ASCE, Vol. 109, 1944, p. 913.

[21] "The Rigid-Frame Bridge," by Arthur G. Hayden and Maurice Barron, John Wiley & Sons, Inc., New York, N. Y., 1950.

[22] Cons. Engr., New York, N. Y.

[23] "Design of a Reinforced Concrete Skew Arch," by Bernard L. Weiner, *Transactions*, ASCE, Vol. 96, 1932, p. 1212.

It would be utterly impossible to publish all such analyses, even if it were not altogether too time-consuming for a practicing engineer to attempt to "dig out" all available numerical data. Such a task should properly be undertaken by one of the numerous college research and experiment stations.

It can merely be stated that, where an extended analysis was made, it corroborated the writer's forecast in the 1932 paper that the two elastic systems were separable. The authors who followed confirmed the findings of the writer rather than the other way around.

As a matter of fact, the separation of the two systems is really of little importance in the arguments presented in this paper, although it is a valuable tool for some purposes. The foregoing answers all of Mr. Hayden's discussion and the first part of Mr. Barron's.

Mr. Barron states that there are three contradictions in the paper, which he identifies as paragraphs 5, 6, and 7:

5. Although the writer states that the expedient of designing on the "skew span" is both incorrect and unnecessary (item 5a) it should be obvious that this expedient does give a rough approximation of the results to be derived from a more logical analysis. Hence the statement referred to as item 5b is not a contradiction. A checker often must resort to the expedient of "roughing out" an approximate result, and such an approximation makes an excellent independent check.

6. Here (item 6a) the question is raised as to why the "total stresses" are computed in an arch or frame while it is stated at the same time (item 6b) that in a "wide structure" such total stresses have very little meaning. This is an interesting point and applies to right structures as well as to skew structures. The answer to this question also anticipates one of Mr. Barron's later points. In the design of arches and frames it is really implicitly assumed that when the structure is loaded with as many trucks (concentrated loads) as it can physically accommodate, the resulting stresses will be greater than the local stresses produced by a very few local loads. Although this is also a study that could very well be undertaken by a research and experiment station, the question is academic to a certain extent since the live-load stresses are only a comparatively small fraction of the dead-load stresses. The latter, of course, are more or less uniformly distributed across the width of the structure. In loading the structure with all the trucks it can carry, line loads are really assumed. In fact, the "equivalent loads" used in modern bridge specifications consist of uniform loads and "line concentrations." Under this type of loading or the line loads of trucks, the total stresses are of value, or "have meaning."

7. The point made here (item 7a) is that the factor b can be factored out of all expressions except the "factor of torsion" (item 7b), and yet it is stated that it is factored out as an arithmetical convenience. The reason for this, as explained in the paper, is that in the "factor of torsion," b appears in the form $\left(\frac{1}{b\,t^3} + \frac{1}{b^3\,t}\right)$; and as the second term is small compared to the first, it can be neglected. The quantity b can thus be factored out, but this does

not mean that the structure can be analyzed in two dimensions. The question still remains whether a stress divided by b represents a stress per unit width. It does so only if the stress in question is uniformly distributed across the width, which is not the case for all stresses.

Mr. Barron states that the writer's 1932 paper advocates that the fascia section be extended for a width of 1.5 times the depth of the section (item 8b) and now advocates a value of 3 instead. However, it is also stated[24] that when b/t is greater than 3, the hyperbolic functions in the St. Venant theory of torsion reach their limiting value. Hence, beyond this distance the torsion stresses are constant to the opposite fascia. At 1.5 times the depth, these functions are substantially constant but not quite. Since in actual pilot designs, it made practically no difference, the writer has been using the larger outside value of three.

The writer did not do what Mr. Barron seems to imply in this part of his discussion (item 9). This brings up the main point of difference between the writer and authors of the various other papers on the subject of skew structures.

No matter what form of equations is used—either "exact" or "approximate"—to obtain the total stresses, it can be assumed for the present purpose that the same results will be obtained with sufficient accuracy for design purposes. The design of the sections—that is, the computation of the stresses in the concrete and the determination of the amount and location of reinforcing steel—brings up the question of stress distribution. It is on this question that there is disagreement. In his discussion of the paper by Mr. Hodges, the writer raised this question of stress distribution and stated that a thorough knowledge of the distribution of the total stresses was necessary. The writer did not say, as Mr. Barron seems to imply, that a complete and "exact" analysis for the total stresses was necessary for the design of a skew frame.

However, since making this statement, the writer has come to the conclusion that what is needed is a thorough study of the entire subject by a group of engineers who will have sufficient time to do the job completely, to set up design standards based on all the material written to date, and to make additional studies if necessary.

It is the writer's further judgment that, more than anything else, tests are needed to determine the stress distribution across a section. To make such a study for a full-centered arch or high-vaulted rigid frame, based on the mathematical theory of elasticity, seems beyond the scope of present-day knowledge. Some mathematical guides are available, however. The University of Illinois, Engineering Experiment Station, has published studies on skew slabs from which clues might be obtained. In the final analysis, the total stresses that are needed depend on the unit stresses that are found to exist in skew structures and they also depend on how these unit stresses can be obtained from the total stresses. All that can be hoped for in any case is close approximations.

If the writer's recommendations (item 10a) do not "square" with Mr. Lovering's logic as stated by Mr. Barron (item 10b), it is possible that this

[24] "Design of a Reinforced Concrete Skew Arch," by Bernard L. Weiner, *Transactions*, ASCE, Vol. 96, 1932, p. 1259.

logic may be at fault. Although it is theoretically true that thickening the section near the knee will decrease the moments near the middle part of the frame, this process is so slow as to be of little practical value. At least, this has been the writer's experience. The writer fails to follow the logic in the argument that it is the temperature stresses, not the skew angle, that are the controlling factor. Since the temperature stresses increase rapidly with the skew, the writer fails to see the point. However, the problem of revising the structure so as to reduce excessive temperature stresses is a matter to be decided by the designer in each individual case, and no general conclusions seem to be possible.

The question (item 11) of whether the structure can be analyzed in two dimensions or whether it must be analyzed in three dimensions has already been discussed. As already stated herein, theoretically b cannot be factored out, but actually or rather, practically, it can be factored since $1/b^3\,t$ can be neglected. Dividing by b does not give the stresses per unit width if they are not uniformly distributed.

The writer can not agree with Mr. Barron that the effects of the skew could not be determined before his demonstration became available. Also, Mr. Barron's reference to commercially patented structures is unfortunate. The writer does not care to enter into a discussion of this subject.

Mr. Barron is in error in stating that there are few single-span or double-span designs available, based on exact analysis. His reference to eighteen equations must refer to a three-span bridge, not a two-span bridge. There are six equations per span.

Mr. Barron is evidently referring to three pilot designs made under the writer's supervision—two two-span rigid frames and one three-span bridge. The former required twelve equations each, and the latter eighteen, as already stated.

The writer fails to understand the reference to b/t in Mr. Barron's discussion of the ellipse of stress. The ellipse of stress, referred to by the writer, is the standard transformation formulas for finding the stresses on any plane if the stresses on an elementary parallelopiped of dimensions dx, dy, and dz are known. These formulas are so called because they can be put in the form of the ellipse equation. The ellipse of stress also leads to the determination of the principal planes. Since this ellipse of stress refers to stresses at a point, b/t does not enter into it.

The discussion by Mr. Barron of the writer's method of analyzing a single-span right rigid frame is irrelevant. It is specifically stated that a designer may use any method he chooses for making the basic design. It might be emphasized, however, that there are reasons for the setup used, and the writer has found from experience that it has marked value. Placing the loads between sections, for instance, eliminates an error in the summations at the section where the influence load is placed. Oddly enough, it was Mr. Hayden, co-author with Mr. Barron of the book on rigid frames who taught the writer some of the "tricks" which displease Mr. Barron. The objections to the sloping legs, etc., as being unnecessary refinements, are not valid. It is sometimes easier to be "exact" than to be approximate, which is the case here. It requires

more effort to approximate on the basis of an idealized structure than it does to analyze the actual structure.

It was not the writer's intention to treat elastic loads. Neither Mr. Barron nor the writer need bother describing "elastic loads." There are good descriptions of this method in many standard texts on statically indeterminate stress analysis.

As to design of the sections, since the transformation proposed by the writer results in finding a principal plane, the writer fails to see the logic in the statement that shears are being added to compression stresses. In any case, once the stresses are resolved on a plane and the steel placed perpendicular to that plane the design is correct. The only question is whether the original transformation was correctly made. If the steel were placed perpendicular to the abutments, the problem would be different and much more difficult as shown by the writer in his 1932 paper. Fortunately the section normal to the fascia (at the fascia) is a principal plane—as was shown on general principles independent of any and all skew arch theories—and, therefore, the design differs in no way from ordinary designs of reinforced concrete for bending and direct stress.

Mr. Barron's statement that it is impossible to develop the fixing moment in a rigid-frame footing is correct. As Mr. Hayden has shown, it makes very little difference in most of the structures whether the footings for the rigid frame are considered fixed or hinged. Hence, if the frame is designed so that it is safe for both conditions, it follows that it is safe for the true condition of partial fixity. Since there is no loss in economy in this method of design, there is no need to find the actual degree of fixity which is equivalent to stating that there is no need to find the actual eccentricity of the thrust. Since it is easier, the analysis is usually made on the basis of the hinged condition, and there is no need for more than a rough approximation of the fixing moment.

Mr. Westergaard's equations apply exactly as Mr. Barron states they do. Since the writer finds that transverse steel is not needed for the design of the dead loads and line-load live load stresses, the only transverse steel needed is for distributing actual wheel loads. Therefore, the problem is the same in a right arch as in a skew arch. The writer has simply suggested a crude method for roughing out the amount of transverse steel needed. If there is a better method even for a right arch, the writer would like to know about it.

Finally, if the writer's contention is correct and the fascia design used for the entire width is both safe and economical, only a right arch-analysis is necessary.

It is to be hoped that some college research and experiment station will undertake this problem, make some tests on stress distribution, review all that has been written on the subject, and finally set up a set of recommendations so that skew arch design will be put on a rational basis that can be accepted by the profession.

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